Theory of probability and random graphs – HW9

Q1. Solution:

$$\mathrm{E}[total\ number\ of\ monochromatic\ copies\ of\ K_k\ for\ 2-coloring\ the\ edges\ of\ K_n] = 2 * \binom{n}{k} 2^{-\binom{k}{2}} = \binom{n}{k} 2^{1-\binom{k}{2}}$$

we remove 1 vertex from each above monochromatic copies of K_k, we will have a $K_x: x = n - \binom{n}{k} 2^{1-\binom{k}{2}}$. And there are no monochromatic copies of K_k.

Q2. Solution:

Use the derandomization technique mentioned in the class. Choose each assignment that makes total number of monochromatic copies below its expectation.

Algorithm 1 find monochromatic K_4

$edges := x_1, x_2, \ldots, x_m$
$color\ choices :=\ v_k \in \{0,1\}$
for $k = 1$ **to** m **do**

$$x_k = \underset{v_k \in \{0,1\}}{\operatorname{argmin}} E[\text{total number of monochromatic } K_4 | x_1 = v_1, \cdots, x_{k-1} = v_{k-1}, x_k = v_k]$$

end for

Q3. Solution:

similar idea

Algorithm 2 permutation σ

$vertices\ V := \{v_1, v_2, \ldots, v_n\}$
$permutated\ vertices\ X :=\ \{x_1, x_2, \ldots, x_n\}$
for $k = 1$ **to** n **do**

$$x_k = \underset{x_k \in V \setminus \{v_1, v_2, \cdots, v_{k-1}, N(v_1), N(v_2), \cdots, N(v_{k-1})\}}{\operatorname{argmax}} E[S(\sigma) | x_1 = v_1, \cdots, x_{k-1} = v_{k-1}, x_k = v_k]$$

end for