Homework of Week 10

Deadline: 9:00am, December 29 (Sunday), 2019

1. Consider a graph in $G_{n,p}$ with $p = c\frac{\ln n}{n}$. Use the second moment method to prove that if $c < 1$ then, for any constant $\epsilon > 0$ and for n sufficiently large, the graph has isolated vertices with probability at least $1 - \epsilon$.

2. Suppose H is a hypergraph where each edge has r vertices and meets at most d other edges. Assume that $d \leq 2^{r-3}$. Prove that H is 2-colorable, i.e. one can color the vertices in red or blue so that no monochromatic edges exist.

3. Use the Lovász Local Lemma to show that, if

$$4\binom{k}{2}\binom{n}{k-2}2^{1-\binom{k}{2}} \leq 1,$$

then it is possible to color the edges of K_n with two colors so that it has no monochromatic K_k subgraphs. Note that this is better than the result obtained by counting.

4. Let $G = (V, E)$ be an undirected graph and suppose each $v \in V$ is associated with a set $S(v)$ of $8r$ colors, where $r \geq 1$. Suppose, in addition, that for each $v \in V$ and $c \in S(v)$ there are at most r neighbors u of v such that c lies in $S(u)$. Prove that there is a coloring of G which assigns to each vertex v a color from $S(v)$ such that, for any edge $(u, v) \in E$, the colors assigned to u and v are different. [Let $A_{u,v,c}$ be the event that u and v are both colored with color c and apply the symmetric Lovász Local Lemma.]

5. Do Bernoulli experiment for 20 trials, using a new 1-Yuan coin. Record the result in a string $s_1s_2...s_i...s_{20}$, where s_i is 1 if the i^{th} trial gets Head, and otherwise is 0.