hw 10

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December 2019

Q1 Solution:

Let X_i denotes i-th vertex is isolated, and $X = \sum X_i$ is the number of isolated vertexes. We have $Pr(X_i = 1) = (1-p)^{n-1}$, $E[X] = E[\sum X_i] = n(1-p)^{n-1}$,

$$\begin{aligned} Var[X] &= \sum\nolimits_{i=1}^{n} Var[X_i] + \sum\nolimits_{i=1}^{n} \sum\nolimits_{j\neq i}^{n} Cov(X_i, X_j) \\ &= n(1-p)^{n-1}(1-(1-p)^{n-1}) + n(n-1)Cov(X_i, X_j) \end{aligned}$$

and

$$\begin{aligned} Cov(X_i, X_j) &= E[X_iX_j] - E[X_i]E[X_j] \\ &= (1-p)^{n-1}(1-p)^{n-2} - (1-p)^{n-1}(1-p)^{n-1} \\ &= p(1-p)^{2n-3}. \end{aligned}$$

According to second moment method, we get

$$Pr(X=0) \leq \frac{Var[X]}{(E[X])^2} = \frac{1}{n(1-p)^{n-1}} - \frac{1}{n(1-p)} + \frac{p}{1-p}$$

The rest will be easy.

Q2 Solution: It can be proved by the Lovász Local Lemma.

Q4 Solution: Note that $Pr(A_{u,v,c}) \leq \frac{1}{64r^2}$ and $d \leq 2(r-1)8r + 8r - 1$.