hw 8

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Q1 Solution:

We have

$$E[|S|] \geq \sum \frac{1}{d(u)+1} \geq \frac{|V|}{D+1} \quad \text{and} \quad p = Pr(X \geq \frac{|V|}{D+1}),$$

and

$$\begin{aligned} E[|S|] &= \sum_{i=1}^{\frac{|V|}{D+1}-1} Pr(|S|=i)i + \sum_{i=\frac{|V|}{D+1}}^{|V|} Pr(|S|=i)i \\ &\leq (1-p)(\frac{|V|}{D+1}-1) + p|V|. \end{aligned}$$

Then, we get

$$p \geq \frac{1}{1+\frac{D|V|}{D+1}}.$$

Using

$$D = \frac{2|E|}{|V|},$$

the rest will be easy.

Q3 Solution:

(1) We use X_i denotes i-th K_4 is monochromatic, and $X = \sum X_i$ denotes the number of monochromatic K_4.Then we have

$$E[\sum_{i=1}^{C_n^4} X_i] = \sum_{i=1}^{C_n^4} E[X_i] = C_n^4 2^{-5}.$$

Combined with $Pr(X \leq E[X]) > 0$, end of proof.

(2)

$$\begin{aligned} E[X] &= \sum Pr(X=i) \cdot i \\ &= \sum_{i=1}^{C_n^4 2^{-5}} Pr(X=i) \cdot i + \sum_{i=C_n^4 2^{-5}+1}^{C_n^4} Pr(X=i) \cdot i \end{aligned}$$